VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

CODE A

JEE ADVANCED BOOSTER TEST-1

JEE 2024

Date	Timing	Maximam Marks	Duration
5th June, 2023	4:00 PM - 7:00 PM	177	3 Hours

General Instructions

1. The question paper consists of 3 Subjects (Subject I: **Physics**, Subject II: **Chemistry**, Subject III: **Mathematics**). Each Subject has **two** sections (Section 1 & Section 2).

2. **Section 1** contains **3 types** of questions **[Type A, Type B and Type C].**

Type A contains **Five (05) Multiple Choice Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

Type B contains **Five (05) Multiple Correct Answers Type Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** is correct.

Type C contains **ONE (01) paragraph**. Based on the paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

3. **Section 2** contains **6 Numerical Value Type Questions**. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the ⊕ sign for positive values. However, for negative values, Θ sign should be bubbled*. (Example: 6, 81, 1.50, 3.25, 0.08)

4. For answering a question, an ANSWER SHEET (OMR SHEET) is provided separately. Please fill your **Test Code, Roll No.** and **Group** properly in the space given in the ANSWER SHEET.

Name of the Candidate (In CAPITALS) : _____

Roll Number : _____

OMR Bar Code Number : _____

Candidate's Signature : _____ **Invigilator's Signature** _____

Syllabus

Physics	:	Simple Harmonic Motion, Wave Motion, Electrostatics
Chemistry	:	Solid state, Theory of Solutions, Chemical Bonding, States of Matter
Mathematics	:	Function, Trigonometry, Quadratic Equation, Binomial Theorem

MARKING SCHEME

SECTION-1 | Type A

- This section contains **Five (05)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the answer. For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

SECTION-1 | Type B

- This section contains **Five (05)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : –2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks; choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get –2 marks.

SECTION-1 | Type C

- This section contains **ONE paragraphs**. Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme;
  Full Marks : +3 If ONLY the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

SECTION - 2

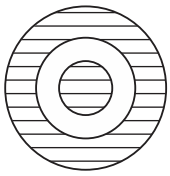
- This section contains **6 Integer Type Questions**. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the ⊕ sign for positive values. However, for negative values, Θ sign should be bubbled*. (Example: 6, 81, 1.50, 3.25, 0.08)
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks: +3 If ONLY the correct Integer value is entered. There is **NO negative marking.**
  Zero Marks: 0 In all other cases.

SUBJECT I : PHYSICS 59 MARKS

SECTION-1 | Type A

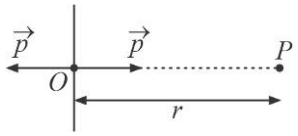
This section consists of 5 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

1. A metal ball of radius R is placed concentrically inside a hollow metal sphere of inner radius $2R$ and outer radius $3R$. The ball is given a charge $+Q$ and the hollow sphere a total charge $-2Q$. The electrostatic potential energy of this system is:



(A) $\dfrac{Q^2}{6\pi\varepsilon_0 R}$ **(B)** $\dfrac{5Q^2}{48\pi\varepsilon_0 R}$ **(C)** $\dfrac{5Q^2}{8\pi\varepsilon_0 R}$ **(D)** $\dfrac{7Q^2}{24\pi\varepsilon_0 R}$

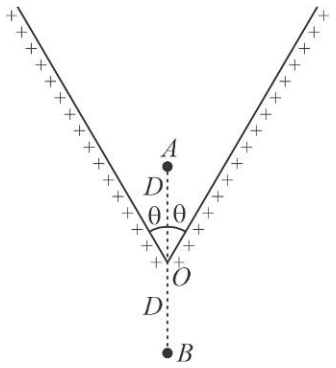
2. Two short dipoles of dipole moment 'p' are kept antiparallel as shown at the origin. Length of each dipole is 'a'. Electric field at point P is:



(A) $\dfrac{6ap}{4\pi\varepsilon_0 r^4}$ **(B)** $\dfrac{3ap}{4\pi\varepsilon_0 r^4}$ **(C)** $\dfrac{3ap}{\pi\varepsilon_0 r^4}$ **(D)** $\dfrac{4ap}{3\pi\varepsilon_0 r^4}$

SPACE FOR ROUGH WORK

3. In the shown figure an infinitely long *V* shaped, uniformly charged wire is placed. Linear charge density on it is λ. The ratio of the electric field intensity (magnitude) at *A* and *B* is: (*AB* bisects the angle and points *A* and *B* are at equal distances from the point *O*).



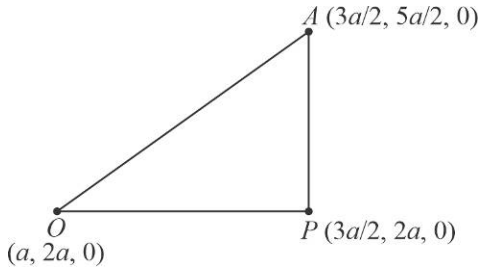
(A) 2

(B) 1

(C) $\cos\left(\frac{\theta}{2}\right)$

(D) $\tan\left(\frac{\theta}{2}\right)$

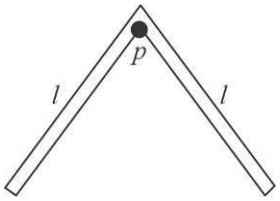
4. A point charge *q* is kept at a point $\left(a, 2a, \frac{a}{2}\right)$ in the space. Consider a triangular plane surface whose co-ordinates of vertices are $(a, 2a, 0), \left(\frac{3a}{2}, 2a, 0\right)$ and $\left(\frac{3a}{2}, \frac{5a}{2}, 0\right)$. The electric flux passing through the considered triangular surface is:



(A) $\frac{q}{12\varepsilon_0}$ **(B)** $\frac{q}{24\varepsilon_0}$

(C) $\frac{q}{48\varepsilon_0}$ **(D)** $\frac{q}{96\varepsilon_0}$

5. A system of two identical rods (*L*-shaped) of mass *m* and length *l* are resting on a peg *P* as shown in the figure. If the system is displaced in its plane by a small angle θ, find the period of oscillations:



(A) $2\pi\sqrt{\frac{4\sqrt{2l}}{3g}}$ **(B)** $2\pi\sqrt{\frac{2\sqrt{2l}}{3g}}$

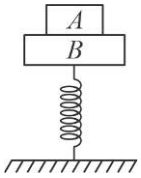
(C) $2\pi\sqrt{\frac{2l}{3g}}$ **(D)** $3\pi\sqrt{\frac{l}{3g}}$

SPACE FOR ROUGH WORK

SECTION-1 | Type B

This section consists of 5 Multiple Correct Answers Type Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** is correct.

6. Block A of mass m is placed over a plank B of mass $5m$. Plank is connected with a spring of force constant K. The system is oscillating. The block A just loses contact with the plank when plank is momentarily at rest. Then choose the correct option(s).



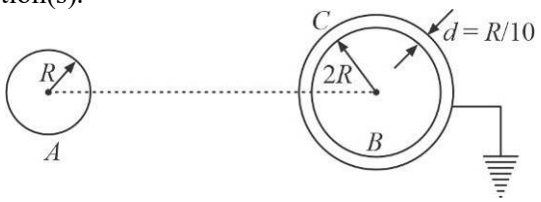
(A) Time period of oscillation is $T = 2\pi\sqrt{\dfrac{6m}{K}}$

(B) Amplitude of oscillation is $A = \dfrac{6mg}{K}$

(C) Maximum compression of spring is $A = \dfrac{12mg}{K}$

(D) Velocity of spring is $\sqrt{\dfrac{16mg}{3K}}$ when its compression is $\dfrac{8mg}{K}$

7. Two conducting spheres A and B of radius R and $2R$ are connected by a thin conducting wire. The conducting sphere 'B' is surrounded by a grounded thin concentric shell 'C' of radius $\left(\dfrac{21}{10}R\right)$. The connecting wire between the sphere A and B is not touching the thin shell 'C'. If conducting sphere 'B' is given a charge 'Q' and the separation between the spheres A and B is much larger than their radii. Then choose the correct option(s).



(A) The charge appearing on the conducting sphere 'A' is $\dfrac{Q}{43}$

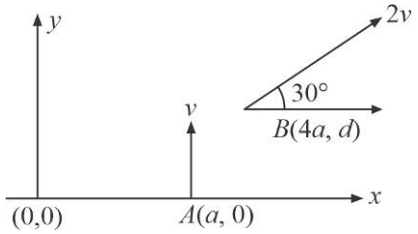
(B) The charge appearing on the conducting sphere 'A' is $\dfrac{Q}{101}$

(C) The charge appearing on the conducting sphere 'B' is $\dfrac{100Q}{101}$

(D) The charge appearing on the conducting sphere 'B' is $\dfrac{42Q}{43}$

SPACE FOR ROUGH WORK

8. A uniform electric field of strength $\vec{E}$ exists in a region. An electron enters a point A with velocity v as shown. It moves through the electric field and reaches at point B. Velocity of particle at B is $2v$ and it is moving at an angle 30° with x-axis shown. Then which of the below option(s) are correct.



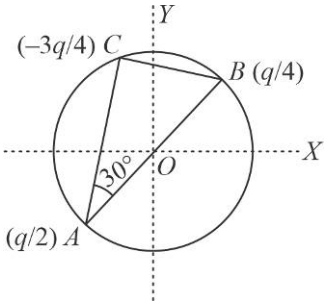
(A) Electric field $\vec{E} = -\frac{mv^2}{2ea}\hat{i} - \frac{mv^2}{8ea}\hat{j}$

(B) Electric field $\vec{E} = \frac{mv^2}{2ea}(-\hat{i})$

(C) Rate of doing work at B is $\frac{(8\sqrt{3}-1)mv^3}{16a}$

(D) Rate of doing work at B is $\frac{\sqrt{3}mv^3}{2a}$

9. Consider a system of 3 charges $q/2$, $q/4$ and $-3q/4$ placed at point A, B and C respectively as shown in the figure. Take O to be the centre of the circle of radius R and $\angle CAB = 30°$. Which of the following option(s) are correct?



(A) The electric field at O is $\frac{\sqrt{13}q}{16\pi\varepsilon_0 R^2}$

(B) Potential energy of the system is zero

(C) Magnitude of force between charges at B and C is $\frac{3q^2}{64\pi\varepsilon_0 R^2}$

(D) Potential at 'O' is zero

SPACE FOR ROUGH WORK

10. The air column in a pipe closed at one end is made to vibrate in its first overtone by a tuning fork of frequency 550 *Hz*. The speed of sound in air is 330 *m*/*s*. End correction may be neglected. Let P_0 denotes the mean pressure at any point in the pipe and ΔP_0 the maximum amplitude of pressure variation in the pipe. Then choose the correct option(s).

(A) The length of the pipe is 45 *cm*

(B) The length of the pipe is 75 *cm*

(C) The maximum pressure at the middle of the pipe is $\left(P_0 + \frac{\Delta P_0}{\sqrt{2}}\right)$

(D) The maximum pressure at the closed end of the pipe is $(P_0 + \Delta P_0)$

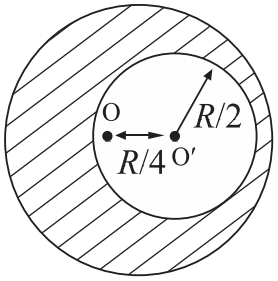
SPACE FOR ROUGH WORK

SECTION-1 | Type C

This section consists of ONE (01) paragraph. Based on each paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

PARAGRAPH FOR Q-11 & 12

A solid sphere having charge density ρ and radius R is shown in figure. A spherical cavity of radius $R/2$ is made in it. The distance between centre of sphere and spherical cavity is $R/4$.



11. The potential at O is:

(A) $\frac{11\rho R^2}{24\varepsilon_0}$ **(B)** $\frac{35\rho R^2}{96\varepsilon_0}$ **(C)** $\frac{13\rho R^2}{248\varepsilon_0}$ **(D)** $\frac{23\rho R^2}{48\varepsilon_0}$

12. The magnitude of electric field at O is:

(A) $\frac{\rho R}{12\varepsilon_0}$ **(B)** $\frac{2\rho R}{15\varepsilon_0}$ **(C)** $\frac{5\rho R}{18\varepsilon_0}$ **(D)** $\frac{\rho R}{21\varepsilon_0}$

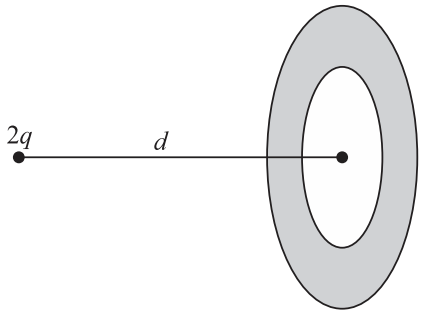
SPACE FOR ROUGH WORK

SECTION-2

This section consists of 6 Numerical Value Type Questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the ⊕ sign for positive values. However, for negative values, Θ sign should be bubbled*. (Example: 6, 81, 1.50, 3.25, 0.08)

1. Three waves $\psi_1 = A\sin[kx-\omega t], \psi_2 = \frac{\sqrt{3}A}{2}\sin(kx-\omega t+\phi)$ and $\psi_3 = 2A\sin(kx-\omega t+\pi/2)$ are superimposed. If resultant wave is given by $\psi = R.\sin(kx-\omega t+\pi/4)$. Where R is some constant. Find ϕ (in degrees). [Take sin 48° = $1/\sqrt{3}$]

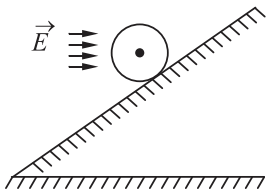
2. A Charge $2q_0$ is placed at a distance of d = 12 *cm* from the centre of annular disc of inner radius 9 *cm* and outer radius 16 *cm*. If flux of electric field through the disc area is $\frac{q_0}{n\in_0}$. Find n?



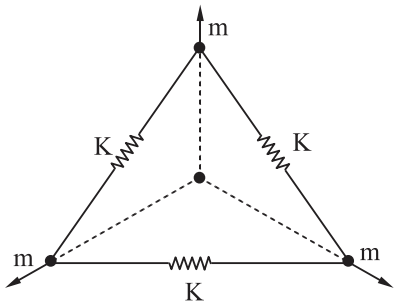
3. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of $2\,ms^{-1}$ in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is $320\,ms^{-1}$, the smallest value of the percentage change required in the length of the pipe is $n/2$. Find the value of n.

SPACE FOR ROUGH WORK

4. A non-conducting uniformly charged sphere is kept on a rough inclined plane. A uniform electric field is present in the horizontal direction as shown in the figure. The angle of inclination is 45° and coefficient of friction between the sphere and the inclined plane is 1/7. As the sphere is released, it is in the state of impending motion and starts pure rolling. For this the ratio of gravitational force to electrostatic force on the sphere is:



5. Three identical springs of force constant k have been joined to the three identical balls (each of mass m), as shown in the figure which are in at the three vertices of an equilateral triangle. In the shown arrangement, each of the spring is in its natural length. When all three balls are simultaneously given small displacements of equal magnitude along the directions (in the plane of triangle) as shown in the figure, the oscillation frequency for the balls will be $\frac{1}{2\pi}\sqrt{\frac{\beta k}{2m}}$ then $\beta =$ _____.



6. The speed of sound in hydrogen is 1270 m/s. The speed of sound (in m/s) in a mixture of oxygen and hydrogen with no. of moles in ratio 1 : 14 at same temperature is $\frac{1270}{\sqrt{n}} m/s$ Find n.

SPACE FOR ROUGH WORK

SUBJECT II : CHEMISTRY 59 MARKS

SECTION-1 | Type A

This section consists of 5 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

1. What is the mass of N_2 in a 15 L gaseous mixture at 20°C and 740 mm pressure. If the composition of the mixture by volume is: $H_2 = 10\%, O_2 = 20\%$ and $N_2 = 70\%$.

(A) 11.90 g **(B)** 16.2 g **(C)** 21.91 g **(D)** 28.00 g

2. C_{60} (buckyball) is cubic closest packed (face-centred cubic) in its crystalline form. If you insert sodium atoms into all the tetrahedral and octahedral holes of the C_{60} structure, the formula would become Na_xC_{60}. What is the value of x?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

3. Lithium Borohydride crystalises in an orthorhombic system with four molecules per unit cell. The unit cell dimensions are a = 6.8 Å, b = 4.4 Å and C = 7.2 Å. If the molar mass is 21.76 g/mol, then the density of crystals is:

(A) 0.67 g/cc **(B)** 1.67 g/cc **(C)** 2.67 g/cc **(D)** 14.5 g/cc

4. Dry air is passed through a solution containing 10 g of the solute in 90 g of water and then through pure water. The loss in weight of the solution is 2.5 g and that of pure solvent is 0.05 g. Calculate the molecular weight of the solute.

(A) 50 **(B)** 180 **(C)** 100 **(D)** 25

5. When a liquid that is immiscible with water was steam distilled at 95.2°C at a total pressure of 748 torr, the distillate contained 1.25 g of the liquid per gram of water. The vapour pressure of water is 648 torr at 95.2°C, what is the molar mass of liquid?

(A) 7.975 g/mol **(B)** 166 g/mol **(C)** 145.8 g/mol **(D)** 14.58 g/mol

SPACE FOR ROUGH WORK

SECTION-1 | Type B

This section consists of 5 Multiple Correct Answers Type Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** is correct.

6. In the depression in freezing point experiment which of the following is/are correct for urea solution in aqueous medium.

(A) The vapour pressure of solution is less than that of pure solvent

(B) Only solvent molecules solidify at the freezing point

(C) The freezing temperature of solution depends upon concentration of solution

(D) The freezing temperature of solution also depends on external pressure

7. A cubic unit cell contains manganese ions at the corners and fluoride ions at the centre of each edge. Which of the following is/are correct?

(A) The empirical formula of the compound is MnF_3

(B) The coordination number of the manganese ion is 6

(C) The coordination number of the fluoride ion is 2

(D) The edge length of the unit cell is 4.02 Å, if the radius of manganese ion is 0.65Å and fluoride ion is 1.36Å

8. For a sample containing O_2 and SO_2 mark out the correct statement(s):

(A) At same temperature average speed of O_2 is greater than that of SO_2

(B) If absolute temperature of O_2 is half to that of SO_2 gas than the speed distribution curves will be different

(C) At same temperature and at lower speed, fraction of molecules of SO_2 will be greater than that of O_2

(D) At same temperature and at higher speed, fraction of molecules of O_2 will be greater than that of SO_2

SPACE FOR ROUGH WORK

9. Benzene and naphthalene form an ideal solution at room temperature. For this process, the true statement(s) is(are).

(A) ΔG is negative

(B) ΔS_{total} is positive

(C) $\Delta H = -ve$

(D) $\Delta S_{surrounding} \neq 0$

10. Equal weights of diborane and ethylene are kept in two vessels of equal volume maintained at T K and 2 T K temperature respectively. Identify the correct statements.

(A) Density of diborane is equal to that of ethylene

(B) Kinetic energy of diborane is greater than that of ethylene

(C) Rate of effusion of diborane is less than that of ethylene

(D) Root mean square velocity of diborane is less than that of ethylene

SPACE FOR ROUGH WORK

SECTION-1 | Type C

This section consists of ONE (01) paragraph. Based on each paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

PARAGRAPH FOR Q-11 & 12

According to V.B.T. atoms of element form bond only to pair up their unpaired electrons present in ground state or excited state. This pairing of unpaired electron will take place by overlapping of orbitals each one having one unpaired electron with opposite spin.

11. Which of the following orbital combinations does not form π- bond?

(A) $p_x + p_x$ sideways overlapping **(B)** $d_{z^2} + p_y$ sideways overlapping

(C) $d_{xy} + d_{xy}$ sideways overlapping **(D)** $d_{yz} + p_y$ sideways overlapping

12. Which of the following can form σ-bond ? (z-axis is internuclear axis).

(A) p_x orbital **(B)** p_y orbital

(C) d_{z^2} orbital **(D)** d_{zx} orbital

SPACE FOR ROUGH WORK

SECTION-2

This section consists of 6 Numerical Value Type Questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the ⊕ sign for positive values. However, for negative values, Θ sign should be bubbled*. (Example: 6, 81, 1.50, 3.25, 0.08)

1. Find the number of correct statements about azeotropic mixture.

(i) When a sizeable deviation from ideality occurs, the boiling point curve for a binary mixture may exhibit a maximum or minimum.

(ii) Mixtures having maximum or minimum boiling point are known as azeotropic mixture.

(iii) These mixtures distil without change in composition.

(iv) It follows that, at a given pressure, the maximum separation of a real solution that can be achieved by fractional distillation is to azeotropic mixture and not for any one of the pure component.

(v) The azeotropic composition may vary significantly with applied pressure.

2. The freezing point of an aqueous solution of KCN containing 0.2 mole/kg water was –0.80°C. On adding 0.1 mole of $Hg(CN)_2$ in the solution containing 1 kg of water, the freezing point of the solution was –0.6°C. Assuming that the complex is formed according to the following equation $Hg(CN)_2 + mCN^- \rightarrow Hg(CN)_{m+2}^{m-}$ and $Hg(CN)_2$ is the limiting reactant, the value of m is_____.

3. The formula of a spinel is $MgAl_2O_4$. How many effective number of anionic vacancy for unit cell will be created if the oxide ions of the spinel are replaced by $X^{-8/3}$ ions?

SPACE FOR ROUGH WORK

4. Consider that AgX crystallizes in rock salt structure. The density of AgX is $6477\ \text{kg/m}^3$ and length of unit cell is 577 pm. Atomic weight of Ag is 108 g/mol. The atomic weight of X (in g/mol, rounded off two decimal places) is: [Take $N_A = 6\times10^{23}$]

5. How many of the following species are linear with zero dipole-moment.

$XeF_2, CO_2, I_3^-, N_2O, NO_2^+, SCN^-, OCN^-, N_3^-$

6. Silver has a cubic unit cell with edge length of 408 pm. If density is $10.6\ \text{g/cm}^3$, how many atoms of silver are there in one unit cell?

SPACE FOR ROUGH WORK

SUBJECT - III : MATHEMATICS 59 MARKS

SECTION-1 | Type A

This section consists of 5 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

1. Let X be the set of all positive integers greater than or equal to 8 and let $f: X \to X$ be a function such that $f(x+y) = f(xy)$ for all $x \geq 4, y \geq 4,$ if $f(8) = 9.$ Then $f(9)$ is equal to:

(A) 19 **(B)** 9 **(C)** 13 **(D)** 8

2. The solution set of $\log_{|\sin x|}(x^2 - 8x + 23) > \dfrac{3}{\log_2 |\sin x|}$ is:

(A) $x \in (3, \pi) \cup \left(\pi, \dfrac{3\pi}{2}\right) \cup \left(\dfrac{3\pi}{2}, 5\right)$ **(B)** $x \in (3, \pi) \cup (\pi, 5)$

(C) $x \in \left(3, \dfrac{5\pi}{2}\right)$ **(D)** $x \in (3, \pi) \cup \left(\pi, \dfrac{3\pi}{2}\right) \cup \left(\dfrac{3\pi}{2}, 5\right) \cup \left(5, \dfrac{5\pi}{2}\right)$

3. A function is defined for all real numbers and satisfies $f(2+x) = f(2-x)$ and $f(7+x) = f(7-x)$ for all real x. If $f(0) = 0,$ what is the least number of roots $f(x) = 0$ must have in the interval $-1000 \leq x \leq 1000.$

(A) 402 **(B)** 401 **(C)** 200 **(D)** 201

4. The sum of all integral values of a values of a where $a \in [-10, 10]$ such that the graph of the function $f(x) = \big||x-2| - a\big| - 3$ has exactly three solutions, is:

(A) 5 **(B)** 13 **(C)** 3 **(D)** 0

5. If $f(x) = 2x^3 - 3x^2 + 1$ then number of distinct real solution(s) of the equation $f(f(x)) = 0$ is:

(A) 0 **(B)** 2 **(C)** 6 **(D)** 3

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SECTION-1 | Type B

This section consists of 5 Multiple Correct Answers Type Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** is correct.

6. Which of the following statement(s) is/are correct?

(A) The coefficient of x^2 in the expansion of $\sum_{r=0}^{100} {}^{100}C_r(x-3)^{100-r}\cdot 4^r$ is equal to 5050

(B) The coefficient of x^2 in the expansion of $\sum_{r=0}^{100} {}^{100}C_r(x-3)^{100-r}\cdot 4^r$ is equal to 4950

(C) If $A_k = \dfrac{{}^nC_k}{{}^nC_k + {}^nC_{k+1}}$ and $\sqrt[3]{\sum_{k=0}^{n-1} A_k} = 4$, then n is equal to 64

(D) If $A_k = \dfrac{{}^nC_k}{{}^nC_k + {}^nC_{k+1}}$ and $\sqrt[3]{\sum_{k=0}^{n-1} A_k} = 4$, then n is equal to 128

7. Let $f(x) = \operatorname{sgn}(\cot^{-1} x) + \tan\left(\dfrac{\pi}{2}[x]\right)$, where $[x]$ is the greatest integer function less than or equal to x. Then which of the following alternatives is/are true?

(A) $f(x)$ is many one but not even function

(B) $f(x)$ is periodic function

(C) $f(x)$ is bounded function

(D) Graph of $f(x)$ remains above x-axis

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8. The equation $\sin(a^2x) = \cos(ax)\ ;\ x \in [0,\ 2\pi]$ has:

(A) 18 solutions if $a = 3$ **(B)** 8 solutions if $a = 2$

(C) 2 solutions if $a = 1$ **(D)** 4 solutions if $a = -1$

9. Let a, b and c are positive real numbers forming an A.P. The equation $ax^2 + 2bx + 5c = 0$ will have real roots for:

(A) $\left|\sqrt{\frac{a}{c}} + \sqrt{\frac{c}{a}}\right| \geq 2\sqrt{5}$ **(B)** $\left|\sqrt{\frac{a}{c}} + \sqrt{\frac{c}{a}}\right| \leq 2\sqrt{5}$

(C) $\left|\sqrt{\frac{a}{c}} - \sqrt{\frac{c}{a}}\right| \leq 4$ **(D)** $\left|\sqrt{\frac{a}{c}} - \sqrt{\frac{c}{a}}\right| \geq 4$

10. The set S of all real x for which $(x^2 - x + 1)^{x-1} < 1$ contains.

(A) $(-5, -1)$ **(B)** $(-1, 1)$ **(C)** $(-1, 0)$ **(D)** $(-3, 1)$

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SECTION-1 | Type C

This section consists of ONE (01) paragraph. Based on each paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

PARAGRAPH FOR Q-11 & 12

If $x, y, z \in R, x+y+z=4, x^2+y^2+z^2=6$. If the range of z is $[p, q]$, then answer the following questions.

11. If $f(x)=\dfrac{q^2x(x^2+1)}{x^2+(x^2+1)^2}$, $x \geq 0$, then range of $f(x)$ is:

(A) $\left[0, \dfrac{1}{5}\right]$ **(B)** $\left[0, \dfrac{8}{5}\right]$ **(C)** $\left[0, \dfrac{5}{8}\right]$ **(D)** $\left[0, \dfrac{12}{5}\right]$

12. Number of integral values of a for which exactly one root of the equation $x^2 + (a + 1)x - 4a = 0$ lies in (p, q) is:

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4

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SECTION-2

This section consists of 6 Numerical Value Type Questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the ⊕ sign for positive values. However, for negative values, ⊖ sign should be bubbled*. (Example: 6, 81, 1.50, 3.25, 0.08)

1. If $(x-y)f(x+y)-(x+y)f(x-y)=4xy(x^2-y^2),$ for all $x, y \in R$ and $f(1)=-2,$ then $f(-5)+f(-4)+f(0)+f(2)+f(3)+f(4)+f(5)-11$ is equal to _____.

2. The number of common points of the curves $y=\sin^{-1}(2x)+\tan^{-1}\left(\frac{1}{[2x]}+2\right)$ and $y=\cos^{-1}(2x+5)+1,$ where $[\cdot]$ denotes the greatest integer function, is _____.

3. If the function $f(x)=\sin(x-1)+\left[\frac{x}{2}\right]+x^3+4x$ and $g(x)=f^{-1}(x),$ then the value of $\left(\frac{1}{g'(5)}\right)$ equals (where $[\cdot]$ represents the greatest integer function).

4. The sum of $\sin^2\frac{2\pi}{7}+\sin^2\frac{4\pi}{7}+\sin^2\frac{8\pi}{7}$ is S, then the value of $4S$ is _____.

5. Let $1+\sum_{r=1}^{10}(3^r\cdot{}^{10}C_r+r\cdot{}^{10}C_r)=2^{10}(\alpha\cdot4^5+\beta)$ where $\alpha,\beta\in N$ and $f(x)=x^2-2x-k^2+1.$ If α,β lie between the roots of $f(x)=0,$ then find the smallest positive integral value of k.

6. If $f(x)=ax^{17}+b\sin x\cdot\sin 2x\cdot\sin 3x+cx^2\operatorname{sgn}(\sin x)+d\log(x+\sqrt{1+x^2})+x(|x+1|-|x-1|)\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)$ be defined on the set of real numbers, $(a>0, b,c,d\in R)$ if $f(-7)=7, f(-5)=-5, f(-2)=3,$ then the minimum number of zeroes of the equation $f(x)=0$ is equal to_____.

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